ELECTROMAGNETIC THEORY.

Electromagnetics is a branch of physics which is used to study electric and magnetic phenomena.

field can be defined as the region in which, at each point there exists a corresponding value of some physical function.

Eg: magnet.

If the field produced due to magnetic effects is called magnetic field.

There are two types of charges +ve and -ve. Such an electric charge produces a field around is called an electric field.

Moving charges produce a current & current carrying conductors produces a magnetic field. In such case electric & magnetic fields are related to each other. Such a field is called electromagnetic field.

The comprehensive study of characteristics of electric, magnetic & combined fields is nothing but the engineering electromagnetics. Such fields may be time varying or time independent.

The distribution of a quantity in a space is defined by a field. Hence to quantify the field, three dimensional representation plays an important role. Such a three dimensional representation can be made easy by the use of vector analysis.

The two quantity which is involved in electromagnetics are : Scalar & vector

Scalar :- is a quantity which has only magnitude & no direction

Eg : temperature, mass, volume etc..

vector :- is a quantity which has both magnitude & direction

Eg :- force, velocity etc..

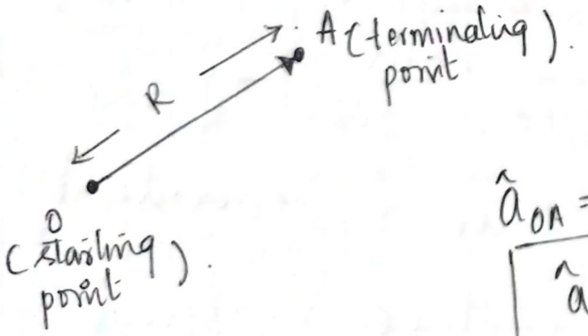
The distribution of a scalar quantity with a definite position in a space is called scalar field.

Eg :- temperature in atmosphere.

If a quantity which is specified in a region to define a field is a vector then it is called vector field.

Eg :- gravitational force on a mass in a space.

Representation of a vector :- In two dimension, a vector can be represented by a straight-line with an arrow in a plane.



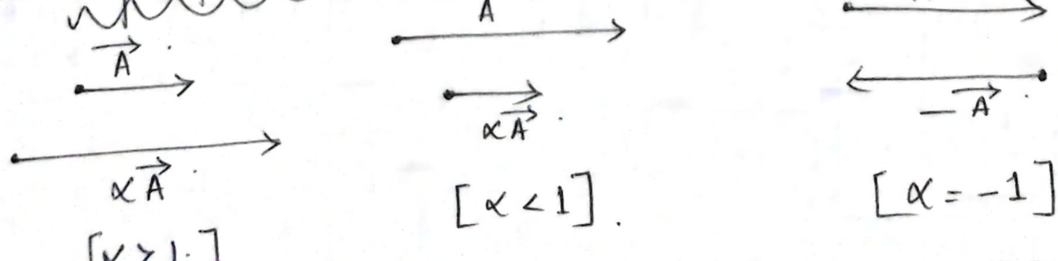
$\vec{OA} = |\vec{OA}|\, \hat{a}_{OA}$.

$|\vec{OA}| = R$ = magnitude of the vector.

$\hat{a}_{OA}$ = unit vector which defines the direction.

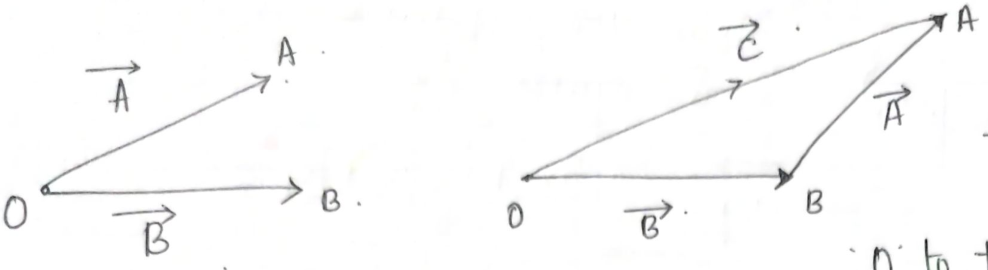
$$\boxed{\hat{a}_{OA} = \frac{\vec{OA}}{|\vec{OA}|}}$$

* Scaling of vector.



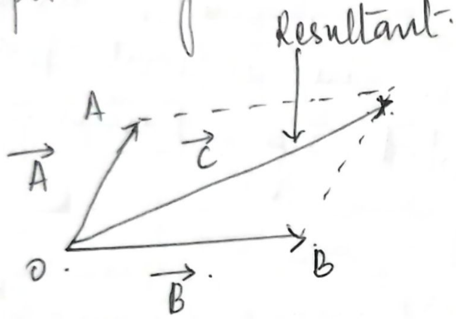
* **Addition of vectors :-** Consider two coplanar vectors. (2)
The vectors which lie in the same plane are called coplanar vectors :

$\vec{C} = \vec{A} + \vec{B}$.

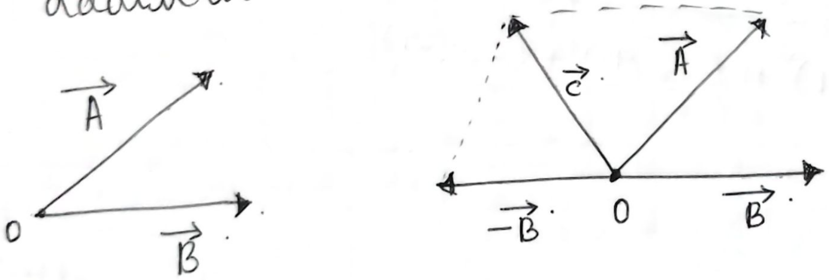


It is remembered that the direction of $\vec{C}$ is from origin O to the tip of the vector moved.

Another method of performing the addition of vectors is the parallelogram rule. The diagonal of the parallelogram represents the addition of two vectors.



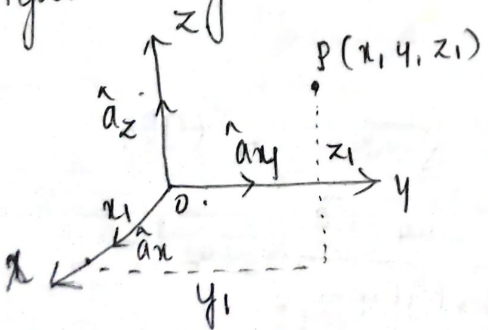
* **Subtraction of vectors :-** is obtained from the rules of addition



$\vec{C} = \vec{A} - \vec{B}$.

* To describe a vector accurately we have three co-ordinate systems.

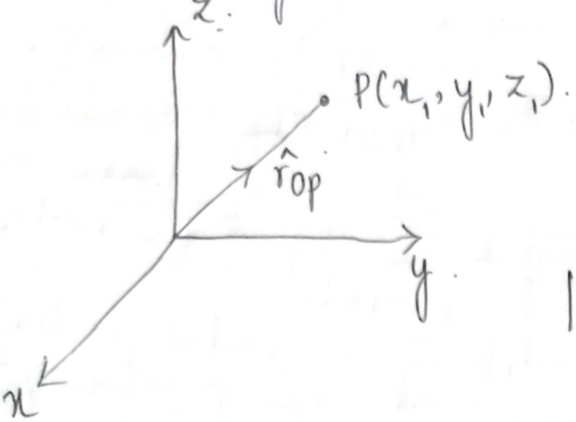
1. **Cartesian co-ordinate system :-** It has three co-ordinate axes represented as x, y and z which are mutually at right angles to each other.



If $x = 0$, then plane indicates two dimensional y-z plane.

Base vectors $\hat{a}_x, \hat{a}_y, \hat{a}_z$ are unit vectors which are strictly oriented along the direction of the co-ordinate axes.

Position vector :- Consider a point $P(x_1, y_1, z_1)$ in a Cartesian co-ordinate system. Then the position vector of point P is represented by the distance of point P from the origin. This is also called radius vector.

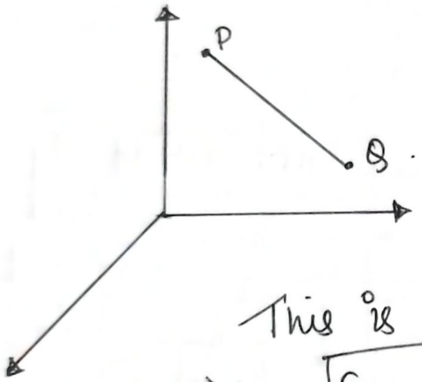


Then the position vector at P is given by

$$\vec{r}_{op} = x_1 \hat{a}_x + y_1 \hat{a}_y + z_1 \hat{a}_z$$

$$|\vec{r}_{op}| = \sqrt{x_1^2 + y_1^2 + z_1^2}$$

Distance vector :- Consider two points in a Cartesian co-ordinate system P & Q with the co-ordinates $P(x_1 y_1 z_1)$ and $Q(x_2 y_2 z_2)$. Then the distance or displacement from P to Q is represented by a distance vector.



$$\vec{PQ} = \vec{Q} - \vec{P}$$

$$= (x_2 - x_1)\hat{a}_x + (y_2 - y_1)\hat{a}_y + (z_2 - z_1)\hat{a}_z$$

This is also called separation vector.

$$|\vec{PQ}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$\hat{a}_{PQ} = \frac{\vec{PQ}}{|\vec{PQ}|}$$

Eg:- Obtain the unit vector in the direction from the origin towards the point $P(3, -3, -2)$.

:- Origin = $(0, 0, 0)$.

$P = (3, -3, -2)$.

$$\vec{OP} = (3-0)\hat{a}_x + (-3-0)\hat{a}_y + (-2-0)\hat{a}_z$$

$$\vec{OP} = 3\hat{a}_x - 3\hat{a}_y - 2\hat{a}_z$$

$$|\vec{OP}| = \sqrt{3^2 + (-3)^2 + (-2)^2} = \sqrt{22} = 4.6904$$

$$\hat{a}_{OP} = \frac{\vec{OP}}{|\vec{OP}|} = \frac{3\hat{a}_x - 3\hat{a}_y - 2\hat{a}_z}{4.6904}$$

$\hat{a}_{OP} = 0.639\hat{a}_x - 0.639\hat{a}_y - 0.426\hat{a}_z$

Eg 2) :- Two points A (2,2,1) and B(3,−4,2) are given in Cartesian system, obtain the vector from A to B & a unit vector directed from A to B.

:- $\hat{a}_{AB} = \dfrac{\vec{AB}}{|\vec{AB}|}$

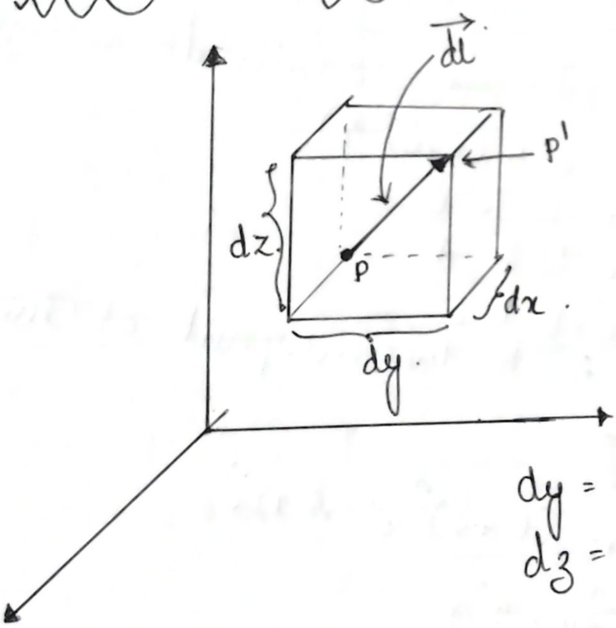
$$\vec{AB} = \vec{B} - \vec{A} = (3-2)\hat{a}_x + (-4-2)\hat{a}_y + (2-1)\hat{a}_z$$

$$\vec{AB} = \hat{a}_x - 6\hat{a}_y + \hat{a}_z$$

$$|\vec{AB}| = \sqrt{1^2 + (-6)^2 + 1^2} = \sqrt{38} = 6.1644$$

$$\hat{a}_{AB} = \frac{\hat{a}_x - 6\hat{a}_y + \hat{a}_z}{6.1644} = 0.162\hat{a}_x - 0.973\hat{a}_y + 0.162\hat{a}_z$$

Differential elements in Cartesian co-ordinate system



Consider a point P(x, y, z) in the rectangular co-ordinate system. Let us increase each co-ordinate by a differential amount. A new point P' will be obtained having co-ordinates (x+dx, y+dy, z+dz).

dx = differential length in X-direction
dy = differential length in Y-direction
dz = differential length in Z-direction.

The distance of P' from P is given by magnitude of the differential vector length.

$$|\vec{dl}| = \sqrt{(dx)^2 + (dy)^2 + (dz)^2}$$

differential vector length is represented as

$$\vec{dl} = dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z$$

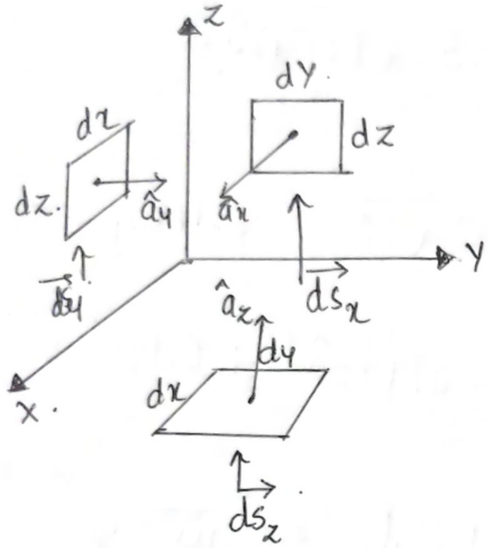
differential volume, $dv = dx\,dy\,dz$.

Elemental Surface :- The differential element ds is represented as,

$$\vec{ds} = ds\,\hat{a}_n .$$

where ds = differential surface area of the element.

$\hat{a}_n$ = unit vector normal to the surface ds.



$\vec{ds_x}$ = differential vector surface area normal to x-direction

$= dy\,dz\,\hat{a}_n$

$\vec{ds_y}$ = differential vector surface area normal to Y-direction.

$= dx\,dz\,\hat{a}_y$

$\vec{ds_z}$ = differential vector surface area normal to z-direction

$= dx\,dy\,\hat{a}_z$

Eg Given three points in Cartesian Co-ordinate system as A(3,-2,1), B(-3,-3,5), C(2,6,-4).

Find i) the vector from A to C.

ii) unit vector from B to A.

iii) the distance from B to C.

iv) the vector from A to the midpoint of the st. line joining B to C.

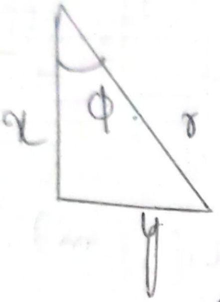
:- i) $\vec{AC} = \vec{C} - \vec{A} = (2-3)\hat{a}_x + (6+2)\hat{a}_y + (-4-1)\hat{a}_z$

$= -\hat{a}_x + 8\hat{a}_y - 5\hat{a}_z$

ii) $\hat{a}_{BA} = \frac{\vec{BA}}{|\vec{BA}|} = \frac{\vec{A}-\vec{B}}{|\vec{A}-\vec{B}|} = \frac{(3+3)\hat{a}_x + (-2+3)\hat{a}_y + (1-5)\hat{a}_z}{\sqrt{6^2+1^2+(-4)^2}}$

$= \frac{6\hat{a}_x + \hat{a}_y - 4\hat{a}_z}{\sqrt{62}} = \frac{6\hat{a}_x + \hat{a}_y - 4\hat{a}_z}{7.2801}$

$\hat{a}_{BA} = \frac{\vec{BA}}{|\vec{BA}|} = 0.8241\hat{a}_x + 0.1373\hat{a}_y - 0.5494\hat{a}_z$



At XY plane,

$\cos\phi = \frac{x}{r}$, $\sin\phi = y/r$.

$\Rightarrow X = r\cos\phi - (1)$, $Y = r\sin\phi - (2)$, $Z = z$.

$eq(1)^2 + eq(2)^2 \Rightarrow x^2 + y^2 = r^2\sin^2\phi + r^2\cos^2\phi$.

$x^2 + y^2 = r^2[\sin^2\phi + \cos^2\phi]$.

$x^2 + y^2 = r^2$

$r = \sqrt{x^2 + y^2}$

$\frac{eq(2)}{eq(1)} \Rightarrow \frac{r\sin\phi}{r\cos\phi} = \frac{Y}{X}$.

$\tan\phi = \frac{Y}{X}$.

$\phi = \tan^{-1}(Y/x)$. , $Z = Z$.

eg. A point P in space is identified with coordinates P(3, 4, 6) mts and another point Q(2, 30°, 4). find
i) the cylindrical co-ordinates of point P.
ii) cartesian co-ordinates of point Q.
iii) find the distance from P to Q.

:- i) P(3, 4, 6)

$x = 3$, $y = 4$, $z = 6$.

$r = \sqrt{x^2 + y^2} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$.

$\tan\phi = y/x \Rightarrow \tan\phi = 4/3 \Rightarrow \phi = \tan^{-1}(4/3) = 53.13$.

$Z = 6$.

ii) Q(2, 30°, 4).

$r = 2$, $\phi = 30°$, $z = 4$.

$X = r\cos\phi = 2\cos 30° = 1.73$.

$Y = r\sin\phi = 2\sin 30 = 1$.

$z = Z = 4$.

iii) $P(3,4,6)$, $Q(1.73, 1, 4)$.

$$|\vec{PQ}| = |\vec{Q} - \vec{P}| = \sqrt{(1.73-3)^2 + (1-4)^2 + (4-6)^2}$$

$$= 3.822 .$$

Eg:- A point is located at $P(4,6,8)$ & $Q(5m, 60^\circ, 3m)$. find

i) cylindrical coordinate of point P.

ii) Cartesian co-ordinate of point Q.

iii) distance from P to Q.

:- Ans i) 7.211 ,

ii) 2.5 ,

iii) 5.5M .

Spherical Co-ordinate System :- Three coordinates at a point P are r, θ, ϕ.

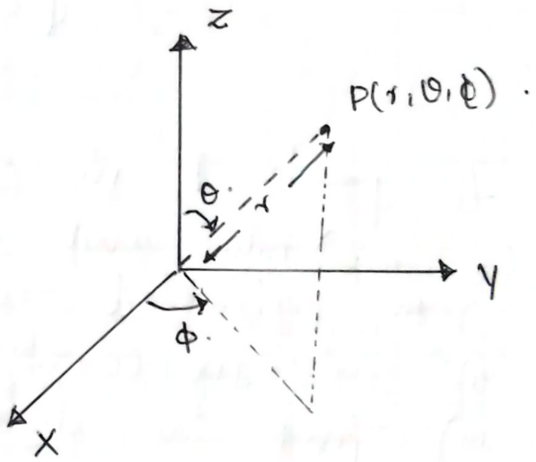
The unit vectors are $\hat{a}_r$, $\hat{a}_\theta$, $\hat{a}_\phi$.

The limits are -

$0 \leq r \leq \infty$.

$0 \leq \theta \leq 180^\circ$

$0 \leq \phi \leq 360^\circ$.



differential vector length

$$\vec{dl} = dr\,\hat{a}_r + r\,d\theta\,\hat{a}_\theta + r\sin\theta\,d\phi\,\hat{a}_\phi$$

$$|\vec{dl}| = \sqrt{(dr)^2 + (r\,d\theta)^2 + (r\sin\theta\,d\phi)^2} .$$

differential volume

$$dv = r^2 \sin\theta\, dr\, d\theta\, d\phi .$$

differential surface .

$$\vec{ds_r} = r^2 \sin\theta\, d\theta\, d\phi\, \hat{a}_r$$

$$\vec{ds_\theta} = r\sin\theta\, dr\, d\phi\, \hat{a}_\theta$$

$$\vec{ds_\phi} = r\, dr\, d\theta\, \hat{a}_\phi$$